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RELIABILITY ANALYSIS OF A SINGLE UNIT SYSTEM BY USE OF EXPONENTIAL DISTRIBUTION SUBJECT TO ARRIVAL TIME OF THE SERVER

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Abstract

In this paper, a single unit system over the redundant systems is considered. The reliability and availability is obtained for a single unit system by use of exponential distribution. Exponential distribution is one of the widely used distribution. The repair activity of the system is carried out by a single server. Regenerative point technique is used to find different reliability measures such as MTSF, reliability and availability of a single unit.

Keywords: Regenerative Point Technique, MTSF, Reliability, Availability and Exponential Distribution

1. INTRODUCTION

While dealing with business and real industrial problem, many researchers have frequently used a single unit system and have contributed in the field of reliability modeling and analysis. Barak A K, Barak M S and Malik S C (2014) checks the feasibility of its repair for a totally failed single unit system. It is replaced by new one when the repair is not feasible. Bashir R, Joorel J P S and Kour R (2016) discussed the inspection policy and analyzed the controlled and uncontrolled demand factor for a single unit system model. Dhankar A K, Bhardwaj, R K and Malik S C (2012) considered complete failure of a single unit system either directly from normal mode or via partial failure. Numerical results are carried out to find the reliability and economic measures. Kumar A, Ram M, Pant S and Kumar A (2018) discussed the different failure with standby of an industrial based complex systems. Malhotra R and Taneja G (2013) investigated the availability and reliability measures of a single unit system depending on demand variation in an industry. Malik S C and Kumar A (2010) considered two reliability model for a single-unit system with a single server and reliability quantities were also derived. Nandal N and Malik S C (2019) use gamma distribution to find the reliability and availability of a single unit system over the redundant system. Nandal N, Grewal A S and Malik S C (2017) evaluated reliability measures using Gamma distribution for a single unit system. Taj S Z, Rizwan S M, Alkali B M, Harrison D K and Taneja G L (2017) carried out reliability measures and maintenance practices for a single machine subsystem of a cable plant using Semi-Markov process and Regenerative Point Technique. Taj S Z, Rizwan S M, Alkali B M, Harrison D K and Taneja G L (2017) analyzed a single machine subsystem of a cable plant with 3 types of maintenance for the subsystem.

Hence, in this paper we have confined our study to a single unit system with exponential distribution for the evaluation of different reliability measures, failure and repair time. To carry out the repair activity of a single unit system, a single server is there to perform repair activity, where the arrival of the server take some time to reach the system. Regenerative point technique is used to carried out some important expressions for reliability. The behavior of reliability measures like MTSF, reliability and availability of a single system is observed for different arbitrary values of the parameters along with numerical results and graphs.

Exponential Distribution

The exponential distribution is the only distribution to have a constant failure rate. Because of its constant failure rate property, the exponential distribution is an excellent model. The probability density function (p d f) of exponential distribution is given by

$f(t) = \lambda e^{-\lambda t}, t \geq 0$

Where $\lambda > 0$ is a scale parameter

The reliability function is defined as

$R(t) = e^{-\lambda t}, \quad t > 0$

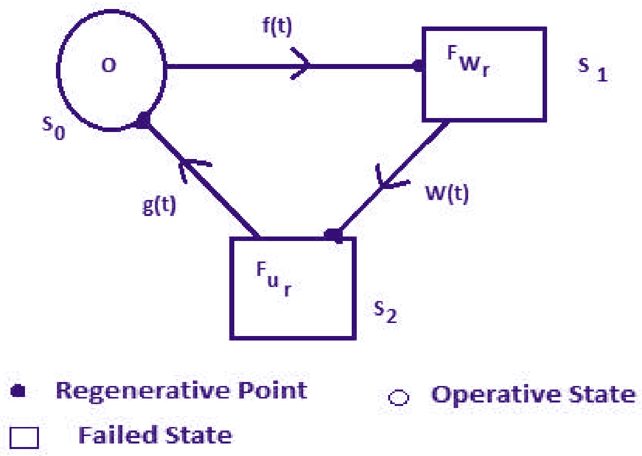
The hazard rate is given by

$h(t) = \lambda, \quad t > 0$

2. SYSTEM DESCRIPTION

System model is shown in Fig. 1

Fig. 1 State Transition Diagram



3. NOTATIONS

O	The unit is operative and in normal mode
F_{u_r}	The system is failed and under repair
F_{w_r}	The system is failed and waiting for repair
S_0	The initial state in which the system is good and operative
S_1	The second state in which system is failed and waiting for repair due to non availability of the server
S_2	The last state in which system is failed and under repair of the server
$g(t)$	Probability Density Function of repair time
$f(t)$	Probability Density Function of failure time
$w(t)$	Probability Density Function of arrival time of the server
$q_{ij}(t)$	Probability density function of first passage time from regenerative state i to a regenerative state j
$Q_{ij}(t)$	Cumulative distribution function of first passage time from regenerative state i to a regenerative state j
p_{ij}	Direct transition probability from state S_i to S_j without passing into any other state
A_0	Availability of the system
m_{ij}	Contribution to the mean sojourn time in state S_i when the system transits directly to state S_j

4. TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

The probabilistic consideration yield the following expressions for the non-zero elements $p_{ij} = \lim_{t\to\infty} Q_{ij}(t) = \int_0^\infty q_{ij}(t)\,dt$

$$dQ_{01}(t) = q_{01}(t)dt = \lambda e^{-\lambda t}dt \quad (1)$$

$$dQ_{12}(t) = q_{12}(t)dt = w(t)dt \quad (2)$$

$$dQ_{20}(t) = q_{20}(t)dt = g(t)dt \quad (3)$$

Taking Laplace Stieltjes Transform of above equations, we get

$$Q_{01}^{**}(s) = \int_0^\infty e^{-st}\, d[Q_{01}(t)] = \int_0^\infty e^{-st}\, \lambda e^{-\lambda t} dt = \frac{\lambda}{(s+\lambda)} \quad (4)$$

$$Q_{12}^{**}(s) = w^*(s) \quad (5)$$

$$Q_{20}^{**}(s) = g^*(s) \quad (6)$$

Taking $s \to 0$, we get the following transition probabilities.

$p_{01} = 1,\ p_{12} = w^*(0) = 1, p_{20} = g^*(0) = 1$

Mean Sojourn Times

The mean Sojourn time in a state is the expected time taken by the system in that state before transiting in to any other state. If T_i be the sojourn time in the state i, then the mean sojourn time in the state I is

$\mu_i = \int_0^\infty \Pr(T_i > t)\ or\ \mu_i = \sum_j m_{ij}$ where $(i = 0,1)$

But $m_{ij} = -\frac{d}{ds}[Q_{01}^{**}(s)]_{s=0}$

$$\text{We have, } m_{01} = -\frac{d}{ds}\left[\frac{\lambda}{(s+\lambda)}\right]_{s=0} = \frac{1}{\lambda} \quad (7)$$

$$m_{12} = -\frac{d}{ds}[w^*(s)]_{s=0} = -w^{*'}(0) \quad (8)$$

$$m_{20} = -\frac{d}{ds}[g^*(s)]_{s=0} = -g^{*'}(0) \quad (9)$$

$$\text{Now, } \mu_0 = m_{01} = \frac{1}{\lambda}, \mu_1 = m_{12} = -w^{*'}(0) \text{ and } \mu_2 = m_{20} = -g^{*'}(0) \quad (10)$$

5. RELIABILITY MEASURES

For the system model, the following reliability measures have been evaluated.

5.1 Mean Time To System Failure (MTSF)

MTSF represents the 'cumulative distribution function of first passage time from regenerative state s_i to a failed state' and is denoted by $\emptyset_i(t)$.

We have

$$\emptyset_0(t) = \emptyset_{01}(t) \quad (11)$$

Taking Laplace Stieltjes Transform of (11), we get

$$\emptyset_0^{**}(s) = Q_{01}^{**}(s) = \frac{\lambda}{(s+\lambda)}$$

$$\text{Now, } MTSF = \lim_{S\to 0}\frac{1-\emptyset_0^{**}(s)}{s} = \lim_{S\to 0}\frac{1-\frac{\lambda}{(s+\lambda)}}{s} = \quad (12)$$

$\left(\frac{0}{0}\right)$ *Indeterminant Form*

On applying L' Hospital Rule, we get

$$MTSF = Q_{01}^{**'}(0) = \mu_0 = m_{01} = \frac{1}{\lambda} \quad (13)$$

5.2 Reliability

The quality or state of being reliable is reliability. Thus, the reliability of the system is defined as

$$R^*(s) = \frac{1-\emptyset_0^{**}(s)}{s} = \frac{1-Q_{01}^{**}(s)}{s} = \frac{1}{(s+\lambda)} \quad (14)$$

The system reliability can be obtained by taking Laplace Inverse of $R^*(s)$, we get

$$R(t) = L^{-1}\left\{\frac{1}{(s+\lambda)}\right\} = e^{-\lambda t} \quad (15)$$

5.3 AVAILABILITY

Availability is the probability that a system will work as required during the period of a mission or is available for use at a specific time "t". The availability ($A_i(t)$) in different states of the system is expressed as

$$\left.\begin{aligned} A_0(t) &= q_{01}(t)© A_1(t) + M_0(t) \\ A_1(t) &= q_{12}(t)© A_2(t) \\ A_2(t) &= q_{20}(t)© A_0(t) \end{aligned}\right\} \quad (16)$$

Taking Laplace Transform of above equations, we have

$$\left.\begin{aligned} A_0^*(s) &= q_{01}^*(s)A_1^*(s) + M_0^*(s) \\ A_1^*(s) &= q_{12}^*(s)\ A_2^*(s) \\ A_2^*(s) &= q_{20}^*(s)\ A_0^*(s) \end{aligned}\right\} \quad (17)$$

On solving equations (17), we get

$$A_0^*(s) = \frac{M_0^*(s)}{[1-(q_{01}^*(s)q_{12}^*(s)q_{20}^*(s))]} \quad (18)$$

The steady state availability is given by

$$A(\infty) = \lim_{t\to\infty} A(t) = \lim_{s\to 0} sA_0^*(s)$$

$$A(\infty) = \lim_{s\to 0} s\left[\frac{M_0^*(s)}{[1-(q_{01}^*(s)q_{12}^*(s)q_{20}^*(s))]}\right]$$

$$A(\infty) = \frac{1}{\left[1-\lambda g^{*'}(0)-\lambda w^{*'}(0)\right]} \quad (19)$$

Now if the repair and arrival time of a server follows an exponential distribution, then we can take

$g(t) = \alpha e^{-\alpha t}$ and $w(t) = \beta e^{-\beta t}$

Then taking Laplace Transform of the above expressions, we get

$$g^*(s) = \int_0^\infty e^{-st} g(t)dt = \frac{\alpha}{(s+\alpha)} \quad \text{and} \quad g^{*'}(s) = \frac{-\alpha}{(s+\alpha)^2}$$

$$w^*(s) = \int_0^\infty e^{-st} w(t)dt = \frac{\beta}{(s+\beta)} \quad \text{and} \quad w^{*'}(s) = \frac{-\beta}{(s+\beta)^2}$$

Now, taking limit $s \to 0$, we have

$g^*(0) = 1$ and $g^{*'}(0) = \frac{-1}{\alpha}$

$w^*(0) = 1$ and $w^{*'}(0) = \frac{-1}{\beta}$

Hence, $A(\infty) = \frac{\alpha\beta}{\alpha\beta+\alpha\lambda+\beta\lambda}$

6. NUMERICAL ILLUSTRATIONS

Fig. 2 MTSF Vs scale parameter λ

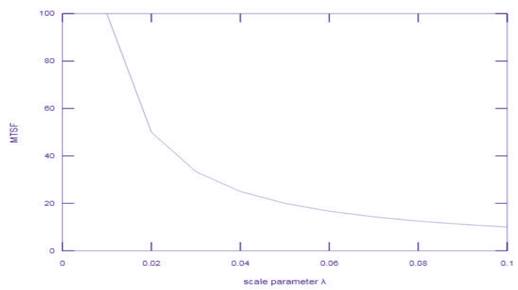


Table 1. MTSF Vs scale parameter

Scale parameter λ	MTSF
0.01	100
0.02	50
0.03	33.333
0.04	25
0.05	20
0.06	16.667
0.07	14.286
0.08	12.5
0.09	11.111
0.1	10

Table. 2 Reliability and scale parameter

	Reliability		
Scale parameter λ	t=10	t=15	t=20
0	1	1	1
0.01	0.9048	0.8607	0.8187
0.02	0.8187	0.7408	0.6703
0.03	0.7408	0.6376	0.5488
0.04	0.6703	0.5488	0.4493
0.05	0.6065	0.4724	0.3679
0.06	0.5488	0.4066	0.3012
0.07	0.4966	0.3499	0.2466
0.08	0.4493	0.3012	0.2019
0.09	0.4066	0.2592	0.1653
0.1	0.3679	0.2231	0.1353

Fig. 3 Reliability Vs scale parameter

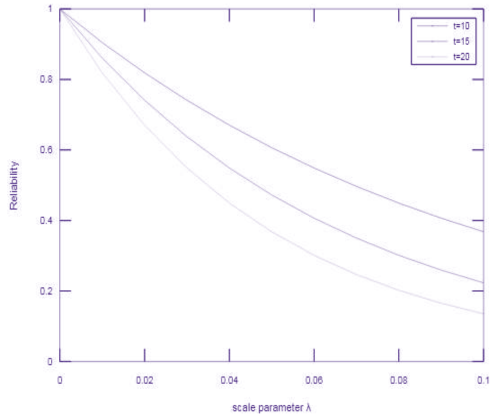


Fig. 4 Availability Vs Scale parameter

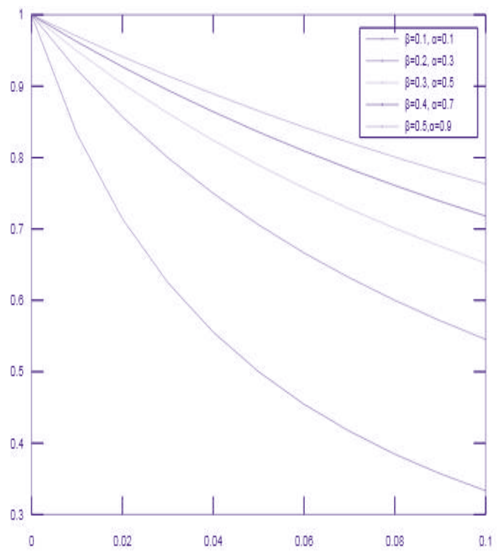


Table. 3 Availability and scale parameter

Scale para meter λ	Availability				
	β=0.1 α=0.1	β=0.2 α=0.3	β=0.3 α=0.5	β=0.4 α=0.7	β=0.5 α=0.9
0	1	1	1	1	1
0.01	0.8333	0.9231	0.9494	0.9622	0.9698
0.02	0.7143	0.8571	0.9036	0.9272	0.9414
0.03	0.625	0.8	0.8621	0.8946	0.9146
0.04	0.5556	0.75	0.8242	0.8642	0.8893
0.05	0.5	0.7059	0.7895	0.8358	0.8654
0.06	0.4545	0.6667	0.7576	0.8092	0.8427
0.07	0.4167	0.6316	0.7282	0.7843	0.8212
0.08	0.3846	0.6	0.7009	0.7609	0.8007
0.09	0.3571	0.5714	0.6757	0.7388	0.7812
0.1	0.3333	0.5455	0.6522	0.7179	0.7627

7. CONCLUSIONS

On the basis of results obtained from table and figure in section 6, the behavior of MTSF, reliability and availability of a single unit system is examined. MTSF decreases as the scale parameter λ increases. With the increase in scale parameter λ and operating time (t), the reliability decreases and on the other hand, availability decreases as the scale parameter λ increases and increases with the increase of arrival time and repair rate.

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